Texas Instruments

MODELING MOTION: High School Math Activities with the CBR

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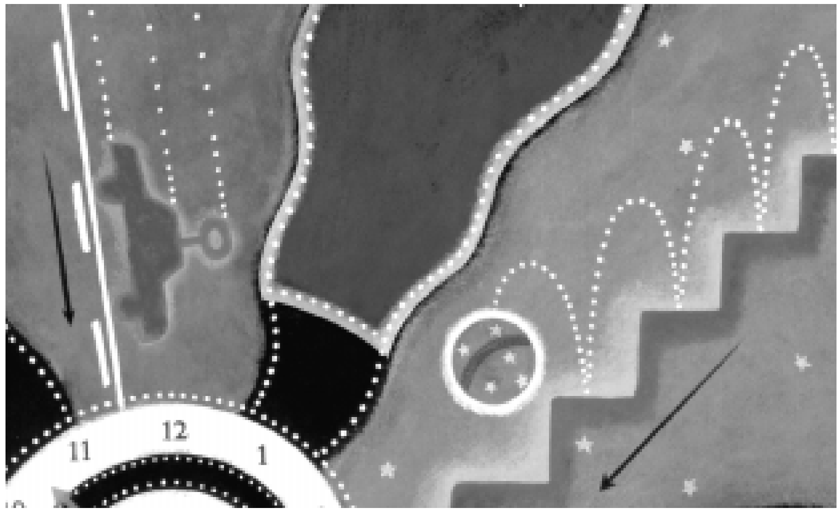

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About the Authors

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SAM GOUGH is currently teaching math and physics at the Kiski School in Saltsburg, PA. Sam has been a national T^3 instructor since 1993 and has taught pre-calculus, calculus and Connecting Math and Science summer institutes. He has co-authored *Discovering Math on the TI-92* and *Work Smarter*, *Not Harder*, a workbook of calculus labs. Sam has been involved with several national math reform movements including the College Board's Pacesetter program, Oregon State's Calculus Connections, and Sam Houston State's Calcnet.

JILL GOUGH is currently a math teacher and Faculty Technology Liason at the Kiski School in Saltsburg, PA. Jill has been a T^3 state instructor since 1994. Since 1996, Jill has been a Phase I and Phase II technology instructor for Mississippi teachers. She has given numerous presentations on the effective use of technology in the classroom (including the calculator, the computer, and the Internet) at local, state and national conferences.

Preface

This workbook provides high school students with “hands-on” motion activities using the Texas Instruments Calculator-Based Ranger™ (CBR™). The CBR™ is a stand-alone motion data collection device that sends data to a TI graphing calculator for analysis. This book provides detailed instructions for analysis on either the TI-83 or TI-82 graphing calculator. The mathematics topics in these activities range from algebra through calculus. Refer to the table on page vi to select activities that emphasize the mathematics topics appropriate for your class.

The book provides 14 different motion activities using the **RANGER** program. This program can be downloaded directly from the CBR™. Each activity has a set of teacher notes that give suggestions for success as well as a set of sample data with solutions to questions. The sample data sets may be entered by hand or downloaded from our web site **www.ti.com/calc**.

Each activity provides directions for analysis using the TI-83 and TI-82. In some cases, the keystrokes differ between the two calculators. Watch for boxed sections and choose the keystrokes appropriate for your calculator.

Many people contributed to this workbook. Some of the activities were adapted from original activities included in *Real-World Math with the CBL System.* Many thanks to the authors Chris Brueningsen, Bill Bower, and Elisa Brueningsen for their support as we wrote this book. Thanks also to the people at Texas Instruments who worked with us on the production of this book, including Jeanie Anirudhan, Scott Webb, Nelah McComsey, and Ron Blasz. Thanks to Michele Jones of ElectroGalley for desktop publishing services, and to Peter Granucci for the chapter illustrations. We appreciate the comments of our students and colleagues who tested activities in their classrooms, especially Susan Boone, who provided a careful review of the first draft.

— Linda Antinone
— Sam Gough
— Jill Gough

Investigation Topics

Activity		Investigation Topics
1	Stretching a Penny	Linear Equation, Direct Variation
2	Flipping a Penny	Linear Equation, Inverse Functions
3	Walk This Walk	Model Distance with a Constant Velocity
4	What is Gravity?	Linear Regression, Extrapolation, Simple Right Triangle Trigonometry
5	How Far Did You Walk?	Area of a Rectangle, $d = rt$
6	Intersection	Linear Equation, Points of Intersection
7	Match This	Piece-wise Linear Functions
8	The Bouncing Ball	Vertex Form of a Quadratic Equation, Linear Equation
9	Conserving Energy	Linear Equation, Quadratic Regression, Symbolic Algebra, Simple Right Triangle Trigonometry
10	Velocity and the Bouncing Ball	Linear Regression, Quadratic Regression, Projectile Motion
11	Bounce Back	Exponential Regression, Linear Equation
12	Sequence of Bounces	Exponential Regression, Geometric Sequence and Series
13	Good Vibrations	Model Sinusoidal Curves
14	Swinging Along	Model Sinusoidal Curves

The RANGER Program

Downloading Instructions

The **RANGER** program is downloaded directly from the CBR to your graphing calculator.

1. Connect the CBR unit and TI-82 or TI-83 graphing calculator with the black calculator-to-CBR cable.
2. On the calculator, press [2nd] **[LINK]** [▸] (the right arrow) to **RECEIVE**, and then press [ENTER].
3. Lift the head of the CBR unit and press the (82/83) key. The program will be transferred to the graphing calculator.

Running the RANGER Program

To initiate the **RANGER** program:

1. Connect the CBR unit and TI-82 or TI-83 graphing calculator with the black calculator-to-CBR cable.
2. Press [PRGM]. Choose **RANGER** and press [ENTER].

This initiates an automatic self-test of the calculator-CBR connection. If the CBR is not properly connected, you are reminded to check the connection; otherwise, the opening screen is displayed. Press [ENTER] and the **MAIN MENU** appears. The **RANGER** program has several built-in menus as shown at right.

1:SETUP/SAMPLE allows the user to select several different options including timing, how the data collection should begin, real-time/non-real-time collection, and smoothing. The real-time data collection feature collects data for 15 seconds and displays this data while it is collected. Non-real-time data collection displays the data afterwards, but allows the user to adjust the total collection time and allows the user to explore the results using distance, velocity and acceleration graphs. When using the real-time collection, only selected data (**DISPLAY** in the set up menu) will be collected.

2:SET DEFAULTS sets the CBR to collect real-time for 15 seconds, to display a **Distance-Time** plot**,** to begin by pressing the [ENTER] key, to apply no smoothing, and to collect and plot in meters.

3:APPLICATIONS contains three useful applications — **Distance Match**, **Velocity Match**, and **Ball Bounce**.

4:PLOT MENU allows you to select the data plot that you would like to view, select plot tools, or repeat the experiment with the same setup. The **PLOT TOOLS** allow you to smooth the data or select a portion of the data for analysis.

5:TOOLS allows for easy sharing of data. (See next section on data sharing.)

6:QUIT takes you out of the program. Before quitting the program, you may trace the data plot, but you cannot use the statistical features of the calculator. You must choose the **QUIT** option before you can analyze the data that has been collected. You must also **QUIT** before data can be shared with another calculator.

Data Sharing

The activity instructions are written to allow every student to collect data using the CBR. The data analysis portion assumes that each student has the data set residing on his or her own calculator. We recognize that it may not be feasible for every student to collect his or her own data due to equipment limitations, time restraints, or class structure. All students can participate after the data collection point if they use the **RANGER** program to share data. The **RANGER** program should be loaded on all student calculators even if the data is collected on only one CBR. Data can be shared either from another calculator or from the CBR. Once the data is received by all calculators using one of the methods outlined below, students may select the desired plot, quit the program, and proceed with the activity instructions.

Receiving Data from the CBR

To receive data from the CBR, use the black calculator-to-CBR cable to connect the calculator and CBR. Run the **RANGER** program on the calculator. Select **5:TOOLS** and then **1:GET CBR DATA**. Follow the onscreen directions (press the [ENTER] key). The data is transferred from the CBR to your calculator. This may be particularly useful if students in a lab group have different types of calculators. The **GET CALC DATA** option discussed next only works when like-models of calculators are connected.

Receiving Data from Another Calculator

Data can only be shared using this method between two like calculators. TI-82s can share data with other TI-82s and TI-83s can share data with other TI-83s, but TI-82s and TI-83s *cannot* share data using the **RANGER** program tools. To receive data from another calculator, the calculator with the data must first **QUIT** the **RANGER** program. Connect the two like calculators together. The calculator with the data must be turned on and be at the home screen. The calculator that will receive the data should run the **RANGER** program. Select **5:TOOLS** and then **2:GET TI83 DATA** (or **2:GET TI82 DATA**). Follow the onscreen directions. The data is transferred from the sending calculator to the receiving calculator.

Name ______________________________

Date ______________________________

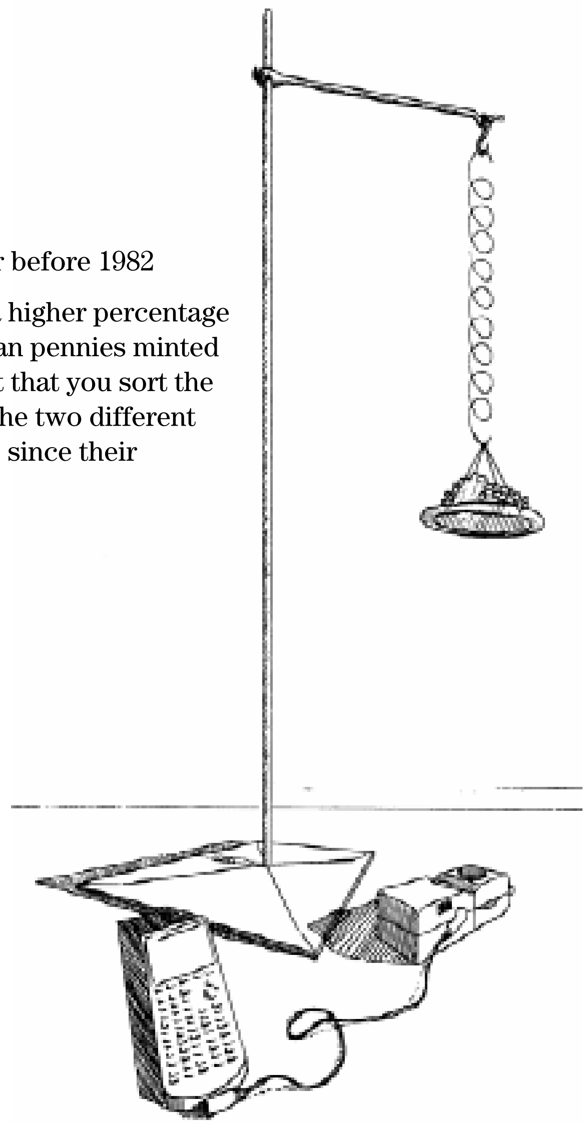
EXPLORATIONS

ACTIVITY 1

Stretching a Penny

Hooke's law states that the force applied to a spring is directly proportional to the distance the spring is stretched, $F \propto x$. The more weight pulling on the spring, the farther the spring stretches.

In this activity, you will investigate how a spring stretches when various weights pull on the spring. You will then relate the stretch of the spring directly to the number of pennies and vice-versa.



You'll Need

- 1 CBR unit
- 1 TI-83 or TI-82 Graphing Calculator
- *BIG* handful of pennies dated after 1983 or before 1982

  Note: Pennies minted prior to 1981 have a higher percentage of copper and thus have a greater mass than pennies minted from 1983 to the present, so it is important that you sort the pennies before you begin and do not mix the two different types of pennies. Do not use 1982 pennies, since their composition varies.
- Spring or slinky
- Paper bowl or plate
- Ring stand or hook

Instructions

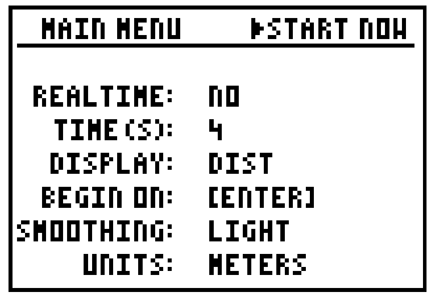
1. Attach the paper bowl or plate to the spring. Hang the spring from the ceiling or a ring stand.

2. Position the CBR face up under the plate.

3. Run the **RANGER** program on your calculator.

4. Enter the setup instructions.

- **a.** From the **MAIN MENU** select **1:SETUP/SAMPLE** to access the setup menu.
- **b.** Press [ENTER] until the **REALTIME** option reads **no**.
- **c.** Press [▼] (the down arrow) to select the next line **TIME (S)** and press [ENTER] [4] [ENTER] to change the time to **4** seconds.
- **d.** Press [▼] to select the next line. Correct or verify the settings and press [ENTER]. Repeat until the options for each line read as shown at right.
- **e.** Press [▼] to move the cursor to the **START NOW** command. Press [ENTER] and follow the directions on the calculator screen.



5. The graph should be a horizontal line. If you are not satisfied with your results, press [ENTER] and select **5:REPEAT SAMPLE**. Trace along the graph to approximate the distance between the plate and the CBR. Record this distance in the data table below.

6. Add 5 pennies to the plate. Press [ENTER] and select **5:REPEAT SAMPLE**.

7. Repeat step 6 until a total of 20 pennies have been added to the plate.

8. Press [ENTER] and select **7:QUIT** to exit the **RANGER** program.

Data Collection

Number of Pennies	Distance to the Plate (meters)
0	
5	
10	
15	
20	

Questions

1. Calculate the total amount of stretch every time 5 pennies is added to the plate by subtracting each distance from the distance when there were no pennies on the plate. Record the stretch in the second column of the table below. This is the stretch measured in meters.

Number of Pennies	Stretch (meters)	Stretch (centimeters)	Stretch per Penny (centimeters/penny)
0			
5			
10			
15			
20			

2. Convert each of the measurements in the second column to centimeters and record it in the third column.

3. Now calculate the amount of stretch per penny by dividing the stretch (the third column) by the total number of pennies on the plate and record the data in the last column of the table.

4. What do you notice about the stretch per penny for each trial?

   ____

5. Grab a big handful of pennies and place them gently on the plate. Trigger the CBR and collect the distance as you did in **Instructions** steps 6, 7, and 8 (page 2). Record this distance.

   Distance in meters: ______________________

   Distance in centimeters: ______________________

6. What would the stretch be in both meters and centimeters for this handful of pennies?

   Stretch in meters: ______________________

   Stretch in centimeters: ______________________

7. Using the information found in question 4, what is the stretch per penny in both meters per penny and centimeters per penny.

   Stretch in meters per penny: ______________________

   Stretch in centimeters per penny: ______________________

8. Predict the number of pennies on the plate:

Number of Pennies = ______________________________

9. Count the number of pennies on your plate. How close was your prediction to the actual number of pennies?

____

10. Suppose that you had 100 pennies on the plate. Calculate the amount of stretch for this handful of pennies in both meters and centimeters. Show how you arrived at this number.

Stretch = ______________________________

11. Suppose that the stretch was 0.5 meters. How many pennies would have been on the plate? Show how you arrived at this number.

Number of Pennies = ______________________________

Make A Mathematical Statement

1. If s is the amount of stretch in centimeters, then show how to predict how many pennies, p, there would be in the plate.

p = ______________________________

2. Since $F \propto x$, then $F = kx$. What would the force, F, be equal to?

____

3. What is the constant of proportionality, k, equal to?

____

4. If p is the number of pennies in the plate, then show how to predict the stretch, s.

s = ______________________________

5. This is also a direct variation. In this case the __________________ is directly proportional to the __________________.

6. What is the proportionality constant for this variation?

k = ______________________________

Name ____________________________

Date ____________________________

EXPLORATIONS

ACTIVITY 8

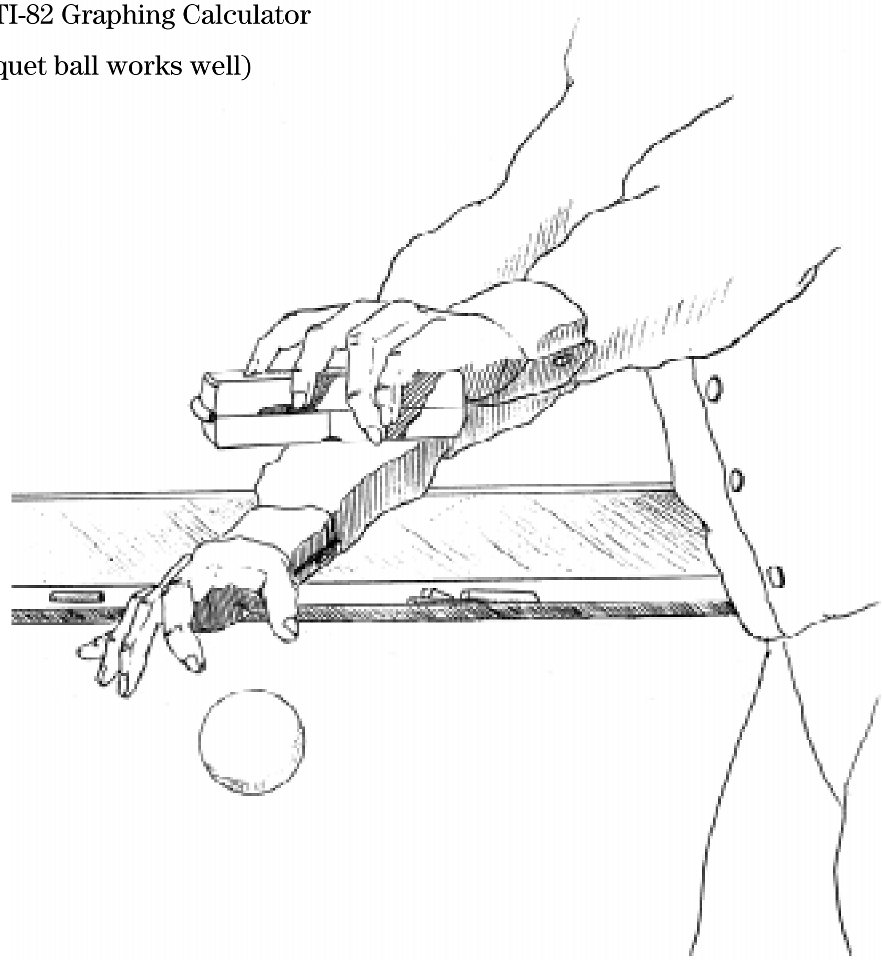
The Bouncing Ball

If a ball is dropped from a given height, what does a Height-Time graph look like? How does the velocity change as the ball rises and falls? What affects the shape of the graphs of both the height and the velocity?

In this activity, you will graph the height of a ball versus time after it is dropped from some height. You will then examine one ball bounce and investigate the parameters that affect the shape of the graph. You will also explore the relationship between the height of the ball and the velocity.

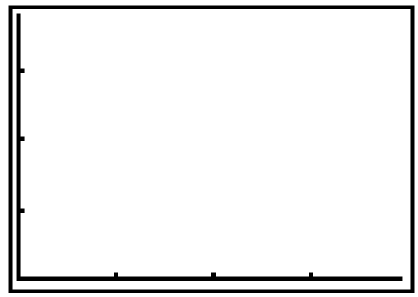
You'll Need

- 1 CBR unit
- 1 TI-83 or TI-82 Graphing Calculator
- Ball (a racquet ball works well)



Instructions

1. Run the **RANGER** program on your calculator.
2. From the **MAIN MENU** of the **RANGER** program, select **3:APPLICATIONS**.
3. Select **1:METERS**, then select **3:BALL BOUNCE**.
4. Follow the directions on the screen of your calculator. Release the ball. Press the (TRIGGER) key on the CBR as the ball strikes the ground.
5. Your graph should have a minimum of five bounces. If you are not satisfied with the results of your experiment, press [ENTER], select **5:REPEAT SAMPLE**, and try again.
6. When you are satisfied with your data, sketch a Distance-Time plot.



Data Collection

1. The goal here is to “capture” one parabola. Use [▸] (the right arrow) to trace to a point near the lower left side of this parabola. Record this point in the **Xmin** category of the table below. Continue tracing until you reach the approximate vertex of the first bounce. Record the coordinates of this point in the table below. Continue tracing until you reach a point near the lower right side of this parabola. Record this point in the **Xmax** category.

	T	***y***
Xmin		
Vertex		
Xmax		

2. Press [ENTER] to return to the **PLOT MENU**. Select **7:QUIT** to exit the **RANGER** program.
3. Press [WINDOW]. Enter the *T* values for **Xmin** and **Xmax** from the table above. Press [GRAPH]. Record the graph of this parabola in the window at the right.

Questions - Part One

1. The vertex form of a parabola is $y = a(x - h)^2 + k$ where h is the x-coordinate and k is the y-coordinate of the vertex of the parabola. Press `2nd` **[QUIT]** to go to the home screen. To store the x-coordinate of the vertex as H, enter the x-coordinate of the vertex, then press `STO▶` `ALPHA` **[H]** `ENTER`. To store the y-coordinate of the vertex as K, enter the y-coordinate of the vertex, then press `STO▶` `ALPHA` **[K]** `ENTER`. To store –1 for A, press `(-)` `1` `STO▶` `ALPHA` **[A]** `ENTER`.

2. Press `Y=`. Enter $A(X - H)^2 + K$ for **Y1**. Press `GRAPH`. A value of –1 for A does not fit this parabola. Press `2nd` **[QUIT]**. Store a better guess for A. Press `GRAPH`. Continue the process of storing a better guess for A until you have fit the parabola.

Record your value for A. ______________

3. What is the equation of the parabola that fits this curve?

y = ______________________________

4. Press `PRGM`. Execute the **RANGER** program. From the **MAIN MENU**, select **4:PLOT MENU**. Select **2:VEL-TIME**.

5. Press `ENTER` to return to the **PLOT MENU**. Select **7:QUIT** to exit the **RANGER** program. Press `WINDOW`. Enter the T-values for **Xmin** and **Xmax** from the table on page 40. Press `GRAPH`. Record the graph of the velocity for this time interval in the window at the right.

6. Press `TRACE`. Record the coordinates of two points that lie on the velocity graph.

Point 1: ______________________________ Point 2: ______________________________

Find the slope and the y-intercept of this line.

m = ______________________________ b = ______________________________

Record the equation of this line.

y = ______________________________

7. Press `Y=`. Enter the equation of this line in **Y2**. Press `GRAPH`. How well does this line fit the velocity data?

____

Questions - Part Two

1. Press [PRGM]. Execute the **RANGER** program. From the **MAIN MENU**, select **4:PLOT MENU**. Select **1:DIST-TIME**.

2. Capture the second parabola.

	T	***y***
Xmin		
Vertex		
Xmax		

3. Press [ENTER] to return to the **PLOT MENU**. Select **7:QUIT** to exit the **RANGER** program. Press [WINDOW]. Enter the *T* values for **Xmin** and **Xmax** from the table above. Press [GRAPH]. Record the graph of this parabola in the window at the right.

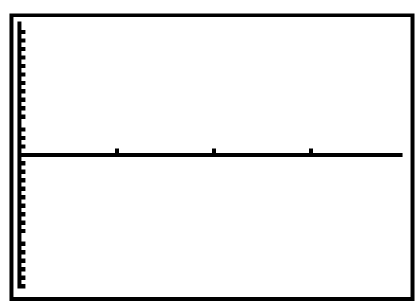
4. Press [2nd] **[QUIT]**. Store the *x*-coordinate of the vertex as H; store the *y*-coordinate of the vertex as K. Store –1 for A. Press [Y=], highlight **Y1**s equal sign and press [ENTER] to activate the function. Press [GRAPH]. Press [2nd] **[QUIT]**. Store a better guess for A. Press [GRAPH]. Continue storing a guess for A until you have fit the parabola.

   Record your value for A. ______________________________

5. What is the equation of the parabola that fits this curve?

   y = ______________________________

6. Press [PRGM]. Execute the **RANGER** program. From the **MAIN MENU**, select **4:PLOT MENU**. Select **2:VEL-TIME**.



7. Press [ENTER] to return to the **PLOT MENU**. Select **7:QUIT** to exit the **RANGER** program. Press [WINDOW]. Enter the *T* values for **Xmin** and **Xmax** from the table above. Press [GRAPH]. Record the graph of the velocity for this time interval in the given window.

8. Record the coordinates of two points that lie on the velocity graph.

   Point 1: ______________________________

   Point 2: ______________________________

9. Find the slope and the y-intercept of this line.

m = ________________________________

b = ________________________________

Record the equation of this line.

y = ________________________________

10. Press [Y=]. Enter the equation of this line in **Y2**. Press [GRAPH]. How well does this line fit the velocity data?

____

Make a Mathematical Statement

1. The equations found in this lab are

Bounce	**Distance-Time**	**Velocity-Time**
1		
2		

2. What influences the change in the ball's velocity?

____

____

3. What is the relationship between the slope of the velocity graph and the leading coefficient, A, of the distance graph?

____

____

4. What force is associated with this quantity?

____

____